# Unit 5: Time value of money

## Contents

* 1. Aims and Objectives
	2. Introduction
	3. Time Value of Money
	4. Techniques of Present Value Vs Future Value
	5. Problems and Solutions
	6. Summary
	7. Answers to Check Your Progress
	8. Model Examination Questions
	9. Reference

**5.0 Aims and objectives**

This unit will discuss the meaning of time value of money, it’s importance in our day-to-day life.

After reading this unit, you will be able to:

* explain the meaning of time value of money
* understand future value and present value
* calculate the future and present values

**5.1 Introduction**

This unit aims at providing basic concepts on the time value of money. This is very important for taking any financial decision. In a business we are investing huge amounts of money today in anticipation of uncertain future returns or revenues. You have already learned that capital is not only scarce but also has cost. Cost in simple terms is nothing but the interest. Suppose you would like to borrow Birr 1, 000 today and return the same after a month without any interest. Do you think some one is going to lend you Birr 1, 000? Definitely no. If you are prepared to pay interest of 3% for a month on the borrowed money, people will come forward to lend you money. The reason is simple money is not available freely and it is capable of earning interest i.e., Birr 30. It is evident that today’s Birr 1, 000 is equivalent to Birr 1, 030 after a month. Here Birr 30 is called cost of capital in financial management.

**5.2 Time value of money**

By experience, we all know that the value of a sum of money received today is more than its value received after some time this is called time value of money. Conversely, the sum of money received in future is less valuable than it is today. The present worth of birr received after some time will be less than a birr received today. Since, a birr received today has more value, individuals, as a rational human beings would naturally prefer current receipt to future receipts.

The time value of money is also known as time preference for money. The time preference for money in business unit normally expressed in terms of rate of return or more popularly as a discount rate. In a business revenues are spread over a period of time i.e., the life of the project. It is nothing but we are trying to calculate the present value versus future value.

**5.3 Techniques of present value Vs future value**

***Compound value:*** where a sum of money deposited one time and earns interest for a specified period. The interest is paid on principal as well as on an interest earned but not withdrawn during earlier period is called compound interest.

 FV = PV + (interest X principal) For example you deposit

 Birr 100 @10% interest

 After one year = 100 + (100 x .10)

 = 110

 After two years = 110 + (110 x .10)

 = 121

 After three years = 121 + (121 x .10)

 = 133.10

 FV1 = P(1+i)

FV2 = P(1+i)2 FV2 = FV1 + F1i P(1+i)2

FV3 = P(1+i)3 FV2 = F1(1+i)P

 FV2 = P

 Fn = P(1+i)n

Here the term (1+i)n is the compounded value factor (CVF) of a lump sum of birr 1. The values may be directly traced from the present value tables. You have already learned the calculation of present value factors in financial accounting II. Hence, you can directly apply the present value factors and find out the values.

The same may be written as below:

 FV = P(CVFn . i)

 FV = Future value

 P = Present value

 CVFn = Compounded value factor year

 i = rate of interest

Suppose you deposit Br. 55, 650 in a bank which will pay you 12 percent interest for a period of 10 years. How much would the deposit grow at the end of ten year?

 FV = P(CVFn . i)

 FV = 55, 650 (CVF10 . 12)

 FV = 55, 650 (3 . 106)

 = 172, 849.90

***Compound value of Annuity:*** An annuity is a fixed payment (or receipt) each year for a specified number of years. Assume that a sum of birr 1 is deposited at the end of each year for four years at 6% interest. This implies that

1(1+.06)3 1.191 Birr 1 deposited at the end of year 1 grow for 3 years.

1(1+.06)2 1.124 Birr 1 deposited at the end of year 2 grow for 2 years.

1(1+.06)1 1.06 Birr 1 deposited at the end of year 3 grow for 1 year.

1. Birr 1 deposited at the end of year 4 grow for no interest.

 4375

 FV4 = A(1+i)3 + A(1+i)2 + A(1+i) + A

 FV4 = A[(1+i)3 + (1+i)2 + (1+i)+1

 FVn = A 

 The same may be written as below

 FV = A(CVAFn i)

 FV = Future value

 A = Annuity

 CVAFn = Compounded Value Annuity Factor to yare

 1 = rate of interest

Assume that you deposit a sum of birr 5, 000 at the end of each year for four years at 6% interest. How much would this annuity accumulate at the end of fourth year.

 FV = A(CVAFn i)

 = 5, 000 (CVAF4 .06)

 = 5, 000 (4.375)

 = 21, 875

***Sinking Fund:*** This is going to be in reverse to the compounded value annuity factor. Here we proceed that to create certain sum of money, how much we have to set aside every year for a specified period.

 FV = A(CVAFn.i)

 A = FV 

 A = FV (SFFn i) SFF = Sinking Fund Factor

 A = F 

For instant to clear off a loan of birr 21, 875 after four years, how much we have to set aside?

 FV = A(CVAFn .1)

 A = FV 

 = 21, 875 FV 

 = 21, 875 x .2286

 = 5, 000

***Present Value:*** here, we calculate the present value of future earnings at a particular rate of interest. This may be further classified into two

1. Present value of a lump sum. The present sum of money to be invested today in order to get birr 1 at the end of year 1, 2, 3 so on and so forth at the rate of 10% interest.

We know F1 = P(1+i) at the end of year 1.

 1 = P (1+10)

 P = 

 

= 0.909

 F2 = P(1+i)2

 1 = P(1+.10)2

 P = 

 = 

 = 0.826

 F3 = P(1+i)3

 1 = P(1+.10)3

 P = 

 =

 = 0.751

 Fn = P(1+I)n

 P = 

 = 

 = Fn [(1+i)n]

You wanted to know the present value of birr 50, 000 to be received after 15 years at the rate of interest 9%

 PV = FV (PVFn i)

 = 50, 000 (PVF15 .09)

 = 50, 000 (.275) present value table

 = 13, 750

***Present value of Annuity:*** An investor some times may receive constant amount for a certain number of years. We may have to calculate the present value of such annuity to be received each year for a specific period.

 

A company receives an annuity of birr 5, 000 for four year at the interest of 10 percent. Then the present value would be

 P = A

 P = 5, 000 Simply, you can refer to the PV tables for PV factor.

 = 5, 000 x 3.170

 = 15, 850

 PV = A(PVAFn .i)

 = 5, 000(3.170) from PV table or using the above formulae

 = 15, 850

***Capital Recovery:*** The reciprocal of the present value annuity factor is called capital recovery factor (CRF). It will give the annuity to repay certain amount of borrowed loan at a particular interest for a specified period.

 PV = A(PVAFn .1)

 A = PV

 A = PV (CRFn .i) CRF = Capital Recovery Factor

 A = P

A company borrows Birr 1, 000, 000 at an interest rate of 15 percent and the loan is to be repaid in 5 equal installments payable at the end of each of the next 5 years. Prepare loan amortization table and the annual installment.

 PV = A (PVAFn .i)

 1, 000, 000 = A (PVAF5 .15)

 1, 000, 000 = A (3.3522)

 = A

 298,311 = A

Loan Amortization Table

**Year Ope. Bala. Annu. Install. Interest Principal Closi. Balance**

 **Biirrs Birrs Birrs Birrs Birrs**

1 1, 000, 000 298, 312 150, 000 148, 312 851, 688

2 851, 688 298, 312 127, 753 170, 559 681, 129

3 68, 129 298, 312 102, 169 196, 143 484, 986

4 484, 986 298, 312 72, 748 225, 564 259, 422

5 259, 422 298, 312 38, 913 259, 399 23

* Birr 23 left because annuity is taken as 298, 312 instead of 298, 311.

***Multi period Compounding:*** Till now, we have seen the cash flows will occur once in a year. But, the cash flows may occur monthly, bi-monthly, quarterly, half yearly and yearly. In such instances we have to apply the following formulae.

 Fn = P

You have deposited a birr of 1, 000 in Commercial Bank of Ethiopia at 12 percent interest per annum. It compound annually, semi-annually, quarterly and monthly for two years. How much does it grow?

1. Annual compounding

n = 2 i = .12%

 FV = P(CVFn .i)

 = 1, 000 (CVF2 .12)

 = 1, 000 (1.254)

 = 1, 254

1. Half-yearly n = 2 x 2 = 4 i = = 6%

FV = 1, 000 (CVF4 .06)

 = 1, 000(1.262)

 = 1, 262

1. Quarters n = 4 x 2 = 8 i = = 3%

FV = 1, 000 (CVF12 .03)

 = 1, 000 (1.267)

 = 1, 267

1. Monthly n = 12 x 2 = 24, i =  = 1%

FV = 1, 000 (CVF24. .01)

 = 1, 000 (1.270)

 = 1, 270

**Check Your Progress –1**

1. If you invest Birr 5, 000 today at a compound interest of 9 percent, what will be its future value after 15 years?

**………………………………………………………………………………………………………………………………………………………………………………………………………………………………………………………………………………………**

1. You want to take a world tour which costs Birr 1, 000, 000, the cost is expected to remain unchanged in normal terms. Your can save Birr 80, 000 annually to fulfill your desire. How long will you have to wait of your savings earn a return of 14 percent annum?

**………………………………………………………………………………………………………………………………………………………………………………………………………………………………………………………………………………………**

**5.4 Problems and solutions**

## Future Value

## *Compound value of Lump sum*

 Future Value = Present value + (Rate of interest) (Present value)

1. You deposit Br. 100 in a bank at 10% interest what would be amount after 3 years.

FV = PV + (PV) (Rate of interest)

 = 100 + 100 (.10)

 = 100 + 10

 = 110

 = 110 + (110) (.10)

 = 110 + 11

 = 121

 = 121 + (121) (.10)

 = 121 + 12.10

 = 133.10

Suppose Br. 1,000 are placed in SBA/C of a Bank at 5% interest what will be the future value after 5 years.

Compound value factor – Table –A

 FV = PV (CVF5 .5)

 = 1,000 (1.266)

 = 1,276

1. If you deposit Br. 55,650 in a Bank at 12% interest for a period of ten years what will be the future value?

FV = 55,650 (CVF10 .12)

 = 55,650 (3.106)

 = 172, 848.90

## *Compound Value of Annuity*

Suppose Br. 1,000 is invested in annuity for four years at the rate of 6%

0 1 2 3 4

 1,000

 1,060

 1,124

 1,191

 4,375

Future sum =

F4 = A (1+I)3 + A(1+I)2 + A(1+I)+A

F4 = A(1+I)3 + (1+I)2 + (1+I)

Fn = A

Fv = A (CVAFn I)

(Compound value factor for annuity factor)

1. You deposit a sum of Br. 5,000 at the end of each year for four years at 6%. How much would this annuity accumulate at the end of the fourth year?

FV = A (CVAF4, .06)

 = 5,000(4.375)

 = 21,875

1. Your father has promised to give you 100,000k, in cash on your 25th birthday. Today is your 16th birthday. He wants to know two things
	1. He decides to make annual payments into a fund after one year, how much will each have to be if the fund pays 8%.
	2. If he decides to invest a lump sum in the account after one year and let it compound annually. How much will the lump sum?
		1. FV = A (CVAFn .I)

100, 000 = A(12.488)

= A

8,007.69 = A

* + 1. FV = PV (CVFn .2)

100,000 = PV (CVF9. .08)

100,000 = PV (1.999)

 = PV

50,025 = P

1. CBE pays 12% interest and compounds quarterly. If Br. 1,000 are deposited initially, how much shall it grow at the end of 5 years?

The quarterly interest will be 3%, number period will be 20

 FV = PV (1+)nxm

 = 1,000 (1.03)20

 = 1,000 x 1.806

 = 1,806

1. How long will it take to double your money if it grows at 12% annually?

FV = PV (CVFn. .12)

Br. 2 = Br. 1 (CVFn .12)

 From Table Br. 2 = CVFn .12) = 1.974 Therefore n = 6

1. Mohan bought a share 15 years age for Br. 10. It is now selling Br. 27.60. What is the compound growth rate the price of share

FV = PV (CVFn .r)

27.60 = 10 (CVF15 .r)

= CVF15 .r)

 From Table 2.760 = 7%

1. X is borrowing Br. 50, 000 to buy a house. If he pays equal installments 25 years and 4% interest on outstanding balances, what is the amount of installment?

 FV = A(CVAFn .r)

50,000 = A (CVAF25 .04)

50,000 = A(15.622)

= A

3,200.61 = A

If it is quarterly payment

 FV = A(CVAF100. .01)

 50,000 = A (63.029)

  = A 793.28 = A

1. A company issued 5,000,000 bonds to be repaid after 7 years. How much should it invest in sinking fund earning 12%, in order to repay bonds.

 FV = A (CFAn .r)

5,000,000 = A (CFA7 .12)

5,000,000 = A (10.089)

= A

495,589 = A

1. X will receive first payment of pension at the end of 10th year form now a Br. of 3,000 a year. The payment will continue for 16 years. How much is the pension worth now, If X’s interest rate is 10%?

Table A-4

 25 years – 9.077

 

 FV = A (CFAn .r)

 = 3,000 (3.318)

 = 9.954

1. A company has to retire Br. 500, 000 debentures of 8% interest 10 years from today. The company plans to put a fixed amount into a sinking fund each year for 10 years. A separate fund is created for the purpose. The first payment will be paid at the end of the current year. The company anticipates that the fund will earn 6% a year. What should be the installments to accumulate 500,000 from now?

Table A-2

 FV = A (CAF10. .06)

 500,000 = A(CAF10. .06 i.e. 13.181)

 = A

 37,933.388 = A

1. A limited company borrows form commercial bank Br. 1,000,000 at 12% interest to be paid in equal annual installments. What would be the size of the installment be? Assume the repayment period is 5 years.

 FV = A (CAF5 .12)

1,000,000 = A (3.605)

= A

277,392.51 = A

1. A sum of Br. 50,000 deposited in a fund which will earn 12% compound semiannually for the first 5 years and 8% interest compounded quarterly for the next 7 years. How much will be amount after 12 years.

First 5 years n = 5 x 2 = 10 Interest = 6%

Second 7 years n = 7 x 4 = 28 interest = 2%

 FV = 50, 000 (CVF10. .06)

 = 50,000 (1.791)

 = 89,542

 FV = 89,542 (CVF14. .02)

 = 89,542 (1.741)

 = 155,895

If A wanted to deposit cash in a saving account at the beginning of year 1, so that he will have Br. 45,000 at the end of 9 years. What is the money to be deposited by A at the beginning of the year 1, if the interest rate for the first five years is 10% compounded semi-annually and the interest rate for the last four years is 12% compounded quarterly?

Last 4% FV = PV = (CVFn .I) = 3%

 45,000 = PV (CVFn 16 .3)

 45,000 = PV (1.605)

 = PV

 28,037 = PV

###### First 5 year

 n = 10 interest =  = 5%

 FV = PV (CVFn.1)

 = PV (CVF10 .05)

 28,037 = PV (1.629)

 = PV

 17,211 = PV

**5.5 Summary**

In our day-to-day life we prefer possession of a given amount of cash now, rather than the same amount at future time. This is time value of money or time preference for money, which arises because of (a) uncertainty of cash flows (b) subjective preference for consumption and

1. availability of investments. The last justification is the most sensible justification for the time value of money.

Interest rate or time preference rate gives money its value and facilitates the comparison of cash flows accounting at different time periods. Two alternative procedures can be used to find the value of cash flows: compounding and discounting. In compounding, future values of cash flows at a given interest rate at the end of a given period of time are found. The future value (F) of a lump sum today (p) for ‘n’ period at ‘i’ rate of interest is given by the following formula:

 Fn = P(1+i)n

 = P(CVFn .i)

The Compound Value Factor (CVFn i) can be found out form the tables.

The future value for annuity for ‘n’ periods at ‘i’ interest may be calculated by the following formula.

 Fn = P

 = P (CVAFn .i)

Compounded Value Annuity Factors (CVAFn .i) is also found from the tables

**5.6 Answers to check Your Progress**

1. Since the table value for 75 years is not available we can take

FV = P(CVFn i)

 = P (CVF30 .0.a) (CVF30 .0.a) (CVF15 . 09)

 = 5, 000(13.268) (13.268) (3.642)

 = 3, 205, 685.1

1. FV = A (CVAFn .i)

1, 000, 000 = 80, 000 (CVAFn . 14)

= CVAFn .14

 12.5

Look into the table for equal or nearest value you will find between 7 and 8 years = 7.72

 or

 1, 000, 000 = 80, 000 (CVAFn .14)

 = 80, 000 

 x 0.14 = 1.14n – 1

 1.75 + 1 = 2.75 = 1.14n

 log 2.75 = n log 1.14

 0.4393 = n x .0569

  = n

 7.72 = years

**5.7 MODEL Examination Questions**

1. What is meant by time value of money?
2. Explain the significance of present and future values in a business organization.